PHYSICAL REVIEW E VOLUME 57, NUMBER 4 APRIL 1998

Electromagnetic modes in cold magnetized strongly coupled plasmas

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(Received 16 December 1997)

The spectrum of electromagnetic waves propagating in a strongly coupled magnetized fully ionized hydrogen plasma is found. The ion motion and damping being neglected, the influence of the Coulomb coupling on the electromagnetic spectrum is analyzed. [S1063-651X(98)12704-6]

PACS number(s): 52.35.Hr, 52.25.Mq, 71.45.Gm

I. INTRODUCTION

The aim of this paper is to find the spectrum of electromagnetic waves propagating in a strongly coupled magnetized fully ionized hydrogen plasma [1], without taking into account the ion motion. We make use of the dielectric tensor of cold magnetized plasmas constructed in Ref. [2] by means of the classical theory of moments.

In neglect of thermal motion this dielectric tensor reads

$$\varepsilon_{\mu\nu}=\begin{pmatrix} \varepsilon_{\perp} & ig & 0 \\ -ig & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\|} \end{pmatrix}, \tag{1}$$

where $\varepsilon_{\perp}$ and $\varepsilon_{\|}$ are the transverse and longitudinal (with respect to the external magnetic field, $\vec{B}$) components of the tensor.

We will consider the damping of the modes in question as negligibly small. This assumption obviously can be verified only experimentally. The damping can be essential, and must be taken into account near the cyclotron resonances. Here the thermal motion of the particles leading to the spatial dispersion must be accounted for also. Thus our results are valid only far from the cyclotron resonances and in coupled plasma systems with the plasma parameter $\Gamma=e^2/aT\gtrsim 1$ ($-e$ is the electron charge, a is the Wigner-Seitz radius, and T is the plasma temperature). For laboratory plasmas this condition implies the temperature $T\sim$ 2–3 eV and the number density of electrons $n\gtrsim 10^{21}$ cm^{-3}. The electrical conductivity σ of such systems with strong Coulomb coupling is of the order of ω_p, so that their effective collisional frequency $\nu=\omega_p^2/4\pi\sigma$ is at least an order of magnitude smaller than ω_p. Similar conditions can also be realized in astrophysical systems (crust of neutron stars, the interior of white dwarfs and large planets with very strong magnetic fields, etc.). Further, we will regard only long wavelength modes for which the condition of a cold plasma [3] holds. In addition, the frequencies of these modes will be presumed to be much higher than the ion cyclotron frequency, so that the ion motion contribution could be neglected.

The components of the dielectric tensor of a system under consideration were found in Ref. [2], and within the first approximation in the ratio $\sqrt{m/M}$ (m and M being the electron and the ion masses)

$$\varepsilon_{\perp}=1-\omega_p^2\frac{\omega^2-\Omega_{\perp}^2}{(\omega^2-\Omega_{\perp}^2)^2-\omega^2\omega_B^2},\quad \varepsilon_{\|}=1-\frac{\omega_p^2}{\omega^2-\Omega_{\|}},$$

$$g=\omega_p^2\frac{\omega\omega_B}{(\omega^2-\Omega_{\perp}^2)^2-\omega^2\omega_B^2}, \tag{2}$$

where $\omega_p=(4\pi ne^2/m)^{1/2}$ is the plasma frequency, and $\omega_B=eB/mc$ is the electron cyclotron frequency. The positive magnitudes $\Omega_{\perp}$ and $\Omega_{\|}$ take into account the Coulomb correlations between the particles, and are expressible via the second frequency moment of the magnetized plasma conductivity tensor Hermitian part [2], so that

$$\Omega_{\perp}^2=\frac{\omega_p^2}{2}\sum_{\vec{q}\neq 0}S_{\rm ei}(\vec{q})\frac{q_{\perp}^2}{q^2},\quad \Omega_{\|}^2=\omega_p^2\sum_{\vec{q}\neq 0}S_{\rm ei}(\vec{q})\frac{q_{\|}^2}{q^2}, \tag{3}$$

$S_{\rm ei}(\vec{q})$ being the partial electron-ion static structure factor, and $q_{\perp}$ (and $q_{\|}$) the projection of the vector $\vec{q}$ on the direction perpendicular (parallel) to the external magnetic field. An analysis of these magnitudes is given in Sec. III; here we mention only that in the ideal plasma limit both $\Omega_{\perp}$ and $\Omega_{\|}\to 0$. We also wish to emphasize that the electron-ion correlations is the factor which guarantees the existence of nonvanishing parameters $\Omega_{\perp}$ and $\Omega_{\|}$. Notice that the above expressions, Eqs. (2) for the dielectric tensor components coincide (within the first order in the ratio $\sqrt{m/M}$) with that of the quasilocalized charges model developed by Kalman and Golden [4].

II. WAVES IN STRONGLY COUPLED MAGNETIZED PLASMAS

If we choose the Cartesian system of coordinates with the z axis parallel to the external magnetic field $\vec{B}$, then the dispersion equation of electromagnetic waves propagating in a magnetized plasma takes the form

$$AN^4+BN^2+C=0, \tag{4}$$

where $N=\omega_0/\omega$ is the scalar refraction index, $\omega_0=|\vec{k}|c$, and

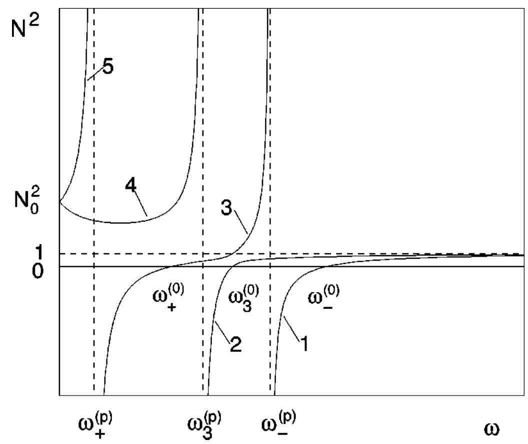

FIG. 1. Squares of refraction indices of strongly coupled magnetized plasma vs frequency (in arbitrary units): (1) fast extraordinary wave, (2) ordinary wave, (3) slow extraordinary wave, and (4,5) strongly coupled plasma whistling sound waves; $(0<\theta<\pi)$.

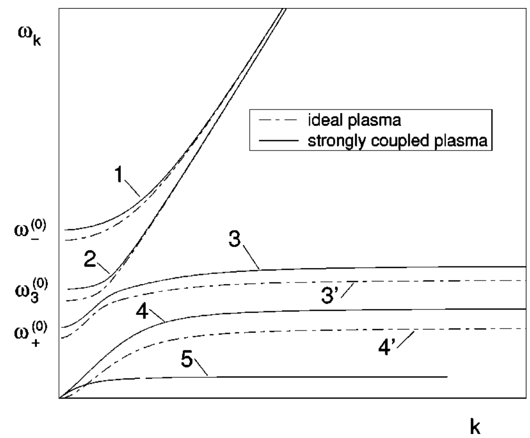

FIG. 2. Frequencies of various eigenmodes of strongly coupled and ideal magnetized plasma vs wave vector (in arbitrary units): (1)–(5) see Fig. 1; (3') slow extraordinary wave of ideal plasma; (4') helicon wave of ideal plasmas $(0<\theta<\pi)$.

$$A=\varepsilon_\perp \sin^2\theta+\varepsilon_\parallel\cos^2\theta,$$

$$B=-\varepsilon_\perp\varepsilon_\parallel(1+\cos^2\theta)-(\varepsilon_\perp^2-g^2)\sin^2\theta,$$

$$C=\varepsilon_\parallel(\varepsilon_\perp^2-g^2). \tag{5}$$

θ is the angle between the wave vector $\vec{k}$ and the magnetic field $\vec{B}$.

Equation (4) has two different solutions,

$$N_\pm^2=[-B\pm(B^2-4AC)^{1/2}]/2A, \tag{6}$$

which are usually associated with ordinary and extraordinary waves: two different kinds of waves of a given frequency and with different refraction indices, which can propagate in magnetized plasmas. These waves are generally elliptically polarized, a wave which propagates along the external magnetic field is transverse polarized; the ordinary wave is characterized by right-handed circular polarization, and the extraordinary wave is left handed polarized.

The frequencies that satisfy the relation $A(\omega,\vec{k})=0$ are traditionally called the plasma resonances. Notice that one of the refraction indices tends to infinity as the frequency approaches the resonance value $N_+^2=-B/A$, while the second one remains finite: $N_-^2=-C/B$. A cubic equation with respect to ω^2 can be obtained from Eq. (5). It determines three resonance frequencies. This is in contrast to ideal magnetized plasmas $(\Omega_\perp=\Omega_\parallel=0)$, where only two resonances exist (we neglect the ion motion). For the case of near longitudinal propagation $\theta\ll 1$, these resonances are

$$\omega_\pm^{(p)}=\frac{1}{2}\{\mp\omega_B+[\omega_B^2+4\Omega_\perp^2]^{1/2}\},$$

$$\omega_3^p=\sqrt{\omega_p^2+\Omega_\parallel^2}\left(1+\frac{\theta^2}{2}\frac{\omega_p^2\omega_B^2}{\omega_p^4-\omega_p^2\omega_B^2-\Omega_\parallel^2\omega_B^2}\right). \tag{7}$$

For the case of transverse propagation we found for the refraction indices poles:

$$(\omega_\pm^{(p)})^2=\Omega_\perp^2+\tfrac{1}{2}\{\omega_p^2+\omega_B^2\mp[(\omega_p^2+\omega_B^2)^2+4\omega_B^2\Omega_\perp^2]^{1/2}\},\quad \omega_3^{(p)}=\Omega_\parallel. \tag{8}$$

The zeros of the $N_\pm^2$ determine the boundaries between the domains of propagation for different waves. From Eq. (4), it follows that $N_\pm=0$, if the coefficient C is equal to zero. We found three zeros,

$$\omega_\pm^{(0)}=\tfrac{1}{2}\{\mp\omega_B+[\omega_B^2+4(\omega_p^2+\Omega_\perp^2)]^{1/2}\},$$

$$\omega_3^{(0)}=(\omega_p^2+\Omega_\parallel^2)^{1/2}. \tag{9}$$

With the poles and zeros determined, and taking into account that $N_\pm^2(\omega=0)=1+\omega_p^2/\Omega_\perp\equiv N_0^2$, and the relation $N_\pm^2(\omega\to\infty)\to 1$, the refraction indices can be plotted. In Fig. 1, the frequency dependence of the refractive indices for an angle $0<\theta<\pi$ is shown.

The branches of propagation $[N^2(\omega)>0]$ are associated with the eigenfrequencies ω_k. The latter are given in Fig. 2 vs wave vector. The modes ω_k are determined by Eq. (4). Since in the present approximation Eq. (4) is the fifth order equation (with respect to ω^2), we find five eigenmodes. In ideal plasmas in neglect of the Alfvén wave only four eigenfrequencies can be found. From Fig. 2, we observe that in the case of a strongly coupled plasma the ideal plasma helicon wave splits into two branches, which we call the strongly coupled plasma whistling sound waves. Thus, in a strongly coupled magnetized plasma, there can exist five eigenmodes: ordinary and extraordinary whistling sound waves, the slow extraordinary, the ordinary and the fast extraordinary waves.

Consider now in more detail the dispersion of the whistling sound waves at small wave numbers, i.e., when $\omega\ll\omega_B$. In this spectral region the dispersion equation reduces to a quadratic equation with respect to ω^2. For the case of mode propagation along the external magnetic field, the corresponding solution reads

$$\omega_k^{(4,5)}=\frac{1}{2}\left\{\pm\frac{\omega_B\omega_0^2}{\omega_p^2+\Omega_\perp^2+\omega_0^2}+\left[\frac{\omega_B^2\omega_0^4}{(\omega_p^2+\Omega_\perp^2+\omega_0^2)^2}+4\frac{\omega_0^2\Omega_\perp^2}{\omega_p^2+\Omega_\perp^2+\omega_0^2}\right]^{1/2}\right\}. \tag{10}$$

In ideal magnetized plasmas (i.e., when $\Omega_\perp=0$) the solution of Eq. (10) then represents the spiral wave—the helicon or the whistler—the frequency of which equals [3]

$$\omega_k^{(h)}=\frac{\omega_0^2\omega_B}{(\omega_p^2+\omega_0^2)}, \tag{11}$$

and tends to zero as $|\vec{B}|\to 0$.

For the case of strong interaction between the particles and at small wave numbers, i.e., when $\Omega_\perp>(\omega_B\omega_0)/\omega_p$, the solutions of Eq. (10) describe the ordinary and extraordinary whistling sound waves propagating in strongly coupled plasmas,

$$\omega_k^{(4,5)}=v_s k\pm\frac{1}{2}\frac{\omega_B\omega_0^2\omega_p^2}{(\omega_p^2+\Omega_\perp^2)^2}, \tag{12}$$

with the whistling sound velocity $v_s=c\Omega_\perp/\sqrt{\omega_p^2+\Omega_\perp^2}$.

The parameter $\Omega_\perp$ will be estimated in Sec. III.

III. ESTIMATE OF THE FREQUENCIES $\Omega_\parallel$ AND $\Omega_\perp$

For our purposes, it is sufficient to make a simple estimate of the frequencies $\Omega_\parallel$ and $\Omega_\perp$ without including their magnetic field dependence, within the random-phase approximation (RPA), and in the hydrogen plasma model. In this approximation they coincide,

$$\Omega_\parallel^2=\Omega_\perp^2=h_{\rm ei}(0)\omega_p^2/3=\omega_p^2\sum_{\vec{q}\neq\vec{0}}S_{\rm ei}(q)/3, \tag{13}$$

and are directly related to [2] the zero separation value of the electron-ion correlation function $h_{\rm ei}(0)$. The electron-ion structure factor can be estimated in a Coulomb system as [5]:

$$S_{\rm ei}(q)=\frac{4\pi e^2}{n\beta q^2}\frac{\Pi_e(q)\Pi_i(q)}{\varepsilon(q)}. \tag{14}$$

Here $\Pi_e(q)$, $\Pi_i(q)$, and $\varepsilon(q)$ are the static electronic and ionic polarization operators (real parts), and the dielectric function, respectively, and β^{-1} is the system temperature in energy units. The ions can be considered as classical particles, and we put $\Pi_i(q)=n\beta$. For $\Pi_e(q)$, we employ a rational interpolation [6]

$$\Pi_e(q)=\frac{\gamma^4/4\pi e^2}{q^2+\gamma^4\lambda_D^2}, \tag{15}$$

constructed to satisfy both long- and short-wavelength limiting conditions of the RPA: $\lambda_D^2=(4\pi e^2 n\beta)^{-1}$ and $\gamma^4=16\pi n e^2 m/\hbar^2$.

After a straightforward calculation, we obtain

$$h_{\rm ei}(0)=2\alpha r_s\times\begin{cases}\dfrac{\sqrt{2}}{\sqrt{B+\sqrt{QS}}+\sqrt{B-\sqrt{QS}}} & \text{if } Q>0\\[2ex] \dfrac{1}{\sqrt{S}} & \text{if } Q\leqslant 0,\end{cases} \tag{16}$$

where $S=B+\sqrt{2A/3}$, and $Q=B-\sqrt{2A/3}$, $A=4\alpha r_s/\pi$, $\alpha=(4/9\pi)^{1/3}=0.521$, and $B=(\pi/3)^{1/3}(A/4\Gamma)+A/6\Theta$. The usual notations are introduced here: k_F is the Fermi wave number; $r_s=a/a_B$ is the Brueckner parameter, i.e., the Wigner-Seitz distance a in the units of the Bohr radius; $\Gamma=\beta e^2/a$, and $\Theta=(\beta E_F)^{-1}$, E_F being the Fermi energy. Notice that $r_s=\Gamma\Theta/0.543$.

In the case of weakly coupled plasmas with $\Gamma\to 0$,

$$h_{\rm ei}(0)\simeq 3.4\Gamma\sqrt{\Theta}\simeq 13.1\sqrt{\frac{1\ \text{eV}}{T}}, \tag{17}$$

with the temperature T measured in units of eV. Equation (16) [or Eq. (17) in the limit of weak coupling] together with Eq. (13) determine the magnitudes $\Omega_\perp$ and $\Omega_\parallel$.

IV. CONCLUSIONS

In this Brief Report the dispersion laws for electromagnetic waves in cold magnetized plasmas are analyzed. Our analysis is based on the expression for the plasma dielectric tensor obtained from the classical theory of moments without using perturbation parameters. Thus both the cases of weak and strong Coulomb coupling are regarded. A qualitative distinction between systems with weak and strong Coulomb coupling is established in their low-frequency electromagnetic wave propagation spectra. It is shown that the weakly coupled plasma helicon branch splits in strongly coupled plasmas into two whistling sound branches. The coupling parameters thermodynamic dependence is estimated.

ACKNOWLEDGMENTS

This work was partly financed by the Deutsche Forschungsgemeinschaft and the Polytechnic University of Valencia, Spain.

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